

Chapter VII



MATHEMATICAL AND LOGICAL REASONING

Learning Objectives

After completion of this unit student will be able to

- Identify mathematically acceptable statements
- express the implications of the compound statements
- validate mathematical statements
- solve logical problems involving syllogism, coding-decoding, odd man out and relationship

Before you start you should know

The fundamentals of algebra and basics of mathematical language studied upto class X.

Concept Map



7.1 Introduction

Deductive reasoning determines whether the truth of a conclusion can be determined, based solely on the truth of the premises. For example: "When it rains, things outside get wet. Thus we can conclude: The grass is outside, therefore: when it rains, the grass gets wet. Mathematical logic and logical reasoning are commonly associated with this type of reasoning. In this unit, we will explore the basics of deductive reasoning such as mathematically acceptable statements, compound statements with 'And' 'Or', 'Not', 'If and only If' and 'If...then'. We will also learn how to check validity of a mathematical statement. Logical functions, Not, If, Or and And are also explored using a spreadsheet as practical work.

Mathematical logic is a subfield of mathematics exploring the applications of formal logic to mathematics. Hence we will also learn different situations to

understand reasoning using logic. This unit also gives the ability to understand the statements using syllogism, to find out relationship using blood relationship, to decode and the ability to classify using odd man out.

7.2 Mathematical Statements

Read the following sentences and classify them:

1. He stopped.
2. Did he stop?
3. I say, stop.
4. How he stopped!

You will notice that these are examples of four types of sentences in English language, classified by their purposes. They are:

1. declarative sentence which is a statement
2. interrogative sentence which is a question
3. imperative sentence which is a command
4. exclamative sentence which is an exclamation

Thus we can see that a statement is a declarative sentence. Declarative sentences tell us something or in other words, they give us information. Now read the following sentences and check whether they are true or false.

1. An apple weighs more than a human being.
2. Those who study mathematics are intelligent.
3. Capital of India is New Delhi.

When we read these sentences, we immediately decide that some sentences are false, some are true and in certain cases we are not sure as they are ambiguous. We find that there is no confusion regarding some sentences. In mathematics sentences that result in a true or a false are called mathematically acceptable statements. Thus the sentence "Capital of India is New Delhi" and "An apple weighs more than a human being" are the mathematically acceptable statement or

mathematical statements. The sentence "Those who study mathematics are intelligent" is an ambiguous statement as intelligence is a relative term and nothing definite can be stated about intelligence of the students who study mathematics.

A sentence is called a mathematically acceptable statement, or simply a statement, if it is either true or false but not both.

While studying mathematics, we come across many such sentences. Some examples are:

1. Two plus two equals four.
2. The sum of two positive numbers is positive.
3. All prime numbers are odd numbers.

Of these sentences, the first two are true and the third one is false. There is no ambiguity regarding these sentences. Therefore, they are mathematical statements. Now consider the following sentence:

The sum of x and y is greater than 0

Here, we are not in a position to determine whether it is true or false, unless we know what are x and y ? For example, it is false when $x = 1$ and $y = -3$ and true when $x = 1$ and $y = 0$. Therefore, this sentence is not a statement. But the sentence: **For any natural numbers x and y , the sum of x and y is greater than 0 is a statement.**

While dealing with statements, we usually denote them by small letters p, q, r, \dots

For example, we denote the statement "Fire is always hot" by p . This is also written as

p : Fire is always hot.

If you wish, It can also be written as

q : Fire is always hot.

Check your Progress-1

Check whether the following sentences are statements. Give reasons for your answer.

1. 8 is less than 6.
 2. Every set is a finite set.
 3. The sun is a star.
 4. Mathematics is fun.
 5. There is no rain without clouds
 6. How far is Chennai from here?
-

7.3 Creating new Statements

We can create new statements from the existing ones by negation or by connecting statements. Let us see how to use negation and make compound statements or split a compound statement to make multiple statements.

a) Using Negation

The denial of a statement is called the negation of the statement. Let us consider the following statement:

p: New Delhi is a city.

The negation of this statement can be in any of the following forms.

- It is not the case that New Delhi is a city
- It is false that New Delhi is a city.
- New Delhi is not a city.

You can very well see that negation of a statement is also a statement which

If p is a statement, then the negation of p is also a statement and is denoted by $\sim p$, and read as 'not p '.

Let

r : There does not exist a quadrilateral which has all its sides equal.

Then negation of this statement will be:

$\sim r$: There exists a quadrilateral which has all its sides equal.

The new statement is true because we know that square is a quadrilateral such that its four sides are equal. Note that negation of a statement can either be true or false. If a statement is true, its negation will be false and if a statement is false, its negation will be true. We can show this fact in a tabular form as given below.

Negation:

Statement A	Statement B as negation of statement A
True	False
False	True

Compound Statements:

Many mathematical statements are obtained by combining one or more statements using some connecting words like "**and**", "**or**", etc.

Compound Statement is a statement which is made up of two or more statements. In this case, each statement is called a component statement.

Let us take an example:

The sky is blue and the grass is green.

This is a compound statement and is made up of two statements p and q.

p: The sky is blue.

q: The grass is green.

p and q are called the component statements. You can also combine these two component statements and make a compound statement using another connecting word, say **or**. Now the compound statement would be:

The sky is blue or the grass is green.

The connecting words which are often found in compound statements are "**and**", "**or**", "**if.... then ...**" "**if and only if**". These are called **connectors**. When we use these compound statements, it is necessary to understand the role of these words

(i) **Compound Statements with 'and'**

Let us take a compound statement: **p: A point occupies a position and its location can be determined.** The statement can be broken into two component statements as :

q: A point occupies a position.

r: Its location can be determined.

Here, we observe that both statements are true. Let us look at another compound statement.

a: 42 is divisible by 5, 6 and 7. This statement has following component statements.

b: 42 is divisible by 5.

c: 42 is divisible by 6.

d: 42 is divisible by 7.

Here, we know that the compound statement a is false, while component statements **c and d** are true. We can conclude the following:

- The compound statement with 'and' is true, if all its component statements are true.
- The compound statement with 'and' is false if any of its component statement is false.

Let us now see whether the above conclusion is true in the following cases.

A line is straight and extends indefinitely in both the directions. The component statements of this compound statement are

p: A line is straight.

q: A line extends indefinitely in both the directions.

Both these statements are true, therefore, the compound statement is true.

For a compound statement: **0 is less than every positive integer and every negative integer;** the component statements are

m: 0 is less than every positive integer.

n: 0 is less than every negative integer.

The second statement is false. Therefore, the compound statement is false.

If the statement is: **All living things have two legs and two eyes;** its two component statements are:

p: All living things have two legs.

q: All living things have two eyes.

Both these statements are false. Therefore, the compound statement is also false.

We can represent these facts in a tabular form as under:

Statement A	Statement B	Compound statement with 'and'
True	True	True
True	False	False
False	True	False
False	False	False

Remember: A statement with a word "and" cannot be always considered as a compound statement

Let us see the following statement.

m: A mixture of alcohol and water can be separated by chemical methods.

This sentence cannot be considered as a compound statement with "and". Here the word "and" refers to two things - alcohol and water and is not used for connecting two statements.

(ii) **Compound Statements with 'or'**

Let us consider the following statement:

p: Two lines in a plane either intersect at one point or they are parallel.

This means that if two lines in a plane intersect, then they are not parallel. Alternatively, if the two lines are not parallel, then they intersect at a point. i.e, this statement is true in both the situations. In order to understand statements with "Or" we first notice that the word "Or" is used in two ways in English language. Let us first look at the following statement.

p: An ice cream or 'lassi' is available with a 'thali' in a restaurant.

This means that a person who does not want ice cream can have a 'lassi' along with a 'thali' or one who does not want 'lassi' can have an ice cream along with a 'thali'. A person cannot have both ice cream and lassi. This is called an exclusive "Or". Here is another statement.

A student who has taken Biology or Chemistry in graduation can apply for M.Sc. Microbiology programme.

Here we mean that the students who have taken both biology and chemistry can apply for the Microbiology programme, as well as the students who have taken only one of these subjects can apply for the Microbiology programme. In this case, we are using **inclusive "Or"**. If we tabulate the outcomes of combining two statements using 'or' in inclusive or exclusive ways, we will get the following:

Statement A	Statement B	Compound statement with 'or' which is exclusive
True	True	False
True	False	True
False	True	True
False	False	False

For **exclusive or** we can state:

1. A compound statement with an 'Or' is true when one of the component statements is true
2. A compound statement with an 'Or' is false when both the component statements are true or both statements are false.

Inclusive or

Statement A	Statement B	Compound statement with 'or' which is inclusive
True	True	True
True	False	True
False	True	True
False	False	False

For **inclusive or** we can state:

1. A compound statement with an 'Or' is true when one component statement is true or both the component statements are true.
2. A compound statement with an 'Or' is false when both the component statements are false.

Check your Progress-2

For each of the following statements, determine whether an inclusive "Or" or exclusive "Or" is used. Give reasons for your answer.

1. The school is closed if it is a holiday or a Sunday.
2. Two lines intersect at a point or are parallel.
3. Students can take French or Sanskrit as their third language.

7.4 Quantifiers

Quantifiers are phrases used in statements like, "**there exists**" and "**for all**". For example, consider the statement.

p: There exists a rectangle whose all sides are equal.

This means that there is at least one rectangle whose all sides are equal. A word closely connected with "there exists" is "for every" (or for all). For example:

p: For every prime number p , \sqrt{p} is an irrational number.

In general, a mathematical statement that says "for every" can be interpreted as saying that all the members of the given set S where the property applies must satisfy that property.

7.5 Implications- 'If-then'

An implication- 'If-then' is the compound statement of the form "if p then q "

Let us consider the statement: **If you are born in some country, then you are a citizen of that country.** When we look at this statement, we observe that it corresponds to two statements p and q given by

p: You are born in some country. q: You are citizen of that country.

Then the sentence "**if p then q** " says that in the event if p is true, then q must be true.

p implies q is denoted by $p \Rightarrow q$. The symbol \Rightarrow stands for implies.

Important aspects of If-then statement:

1. "if p then q" is that, **it does not say anything on q when p is false**. In the given example, if you are not born in the country, then you cannot say anything about q.
2. "if p then q" is that, the statement **does not imply that p happens**. That means if you are a citizen of a country does not imply that you are born in that country.

Contrapositive Statement : A contrapositive statement can be formed from a given statement with "**if-then**". For example, let us consider the following "if-then" statement.

If the physical environment changes, then the biological environment changes.

The contrapositive of this statement is

If the biological environment does not change, then the physical environment does not change.

More examples:

1. If a number is divisible by 9, then it is divisible by 3.
2. If you are born in India, then you are a citizen of India.
3. If a triangle is equilateral, it is isosceles.

The contrapositive of the these statements are

1. If a number is not divisible by 3, it is not divisible by 9.
2. If you are not a citizen of India, then you were not born in India.
3. If a triangle is not isosceles, then it is not equilateral.

Contrapositive of the statement : if p then q is "if \sim q then \sim p"

Converse Statement: A **converse** statement can be formed from a given statement with "if-then". The converse of a given statement "**if p then q**" is "**if q then p**".

Let us take an example. For the statement: **p: If a number is divisible by 10, it is divisible by 5.**

the converse is **q: If a number is divisible by 5, then it is divisible by 10.**

7.6 Implications 'If and only if'

Given below are two pairs of statements.

p: If a rectangle is a square then all its four sides are equal.

q: If all the four sides of a rectangle are equal then the rectangle is a square.

If we combine these two statements using "if and only if" we will get:

r: A rectangle is a square if and only if all its four sides are equal.

'If and only if', represented by the symbol ' \Leftrightarrow ' means the following equivalent forms for the given statements p and q.

- p if and only if q
- q if and only if p
- p is necessary and sufficient condition for q and vice-versa
- $p \Leftrightarrow q$

7.7 Validating Statements

Validating a statement is to check whether a statement is true. Validation of a statement depends upon which of the special words and phrases "and", "or", and which of the implications "if and only if", "if-then", and which of the quantifiers "for every", "there exists", appear in the given statement.

Let us summarize what we have learnt and state as rules for validation. This will be useful in our next section on application of logical reasoning.

Rule 1

Compound statement with 'and'

If p and q are mathematical statements, then in order to show that the statement " p and q " is true, the following steps are followed.

Step-1 Show that the statement p is true.

Step-2 Show that the statement q is true.

Rule 2

Compound Statements with "or" (exclusive or)

If p and q are mathematical statements, then in order to show that the statement " p or q " is true, one must consider the following:

Case 1 By assuming that p is false, show that q must be true.

Case 2 By assuming that q is false, show that p must be true.

Rule 3

Implications "If-then"

Case 1 By assuming that p is true, prove that q must be true. (Direct method)

Case 2 By assuming that q is false, prove that p must be false. (Contrapositive method)

Rule 4

Implications "if and only if "

In order to prove the statement " p if and only if q ", we need to show.

Step-1 If p is true, then q is true.

and

Step-2 If q is true, then p is true.

7.8 Applications of Reasoning

Let us now use the learning about the statements and connectives in developing our reasoning ability. It is expected that you are able to solve questions of reasoning of categorical syllogism, coding-decoding, blood relationship and odd man out. This will help us in development of logical reasoning by enhancing our ability to classify, find patterns and relationship, analyse etc. Theoretical aspects given below are meant just to set the context.

Note to teachers: The assessment should be restricted to problems based on categorical syllogism, coding-decoding, blood relationship and odd-one out. The classroom transactions should focus on the development of logical thinking based on these examples without much focus on the technical terms given in the unit or rote learning of concepts. The explanation is given only for setting the context.

a) Categorical Syllogism

We have learned about mathematical statements and the rules for validation in the earlier part of this unit. Let us make use of this learning in understanding syllogism. Syllogism is a word derived from a Greek word *sylogismos* meaning "conclusion or inference". It is a kind of logical argument that applies deductive reasoning to arrive at a conclusion based on two or more propositions that are asserted or assumed to be true. Categorical syllogisms consist of three categorical propositions or parts as given below:

- Major premise
- Minor premise
- Conclusion

Each of the premises has one term in common with the conclusion. In a major premise, this is the major term (i.e., the predicate of the conclusion). In a minor premise, this is the minor term (i.e., the subject of the conclusion).

For example:

Major premise: All humans are mortal.

Minor premise: All Indians are humans.

Conclusion: All Indians are mortal.

Each of the three distinct terms represents a category. In the above example, humans, mortal, and Indians are three different categories. Mortal is the major term and Indians is the minor term. The premises also have one term in common with each other, which is known as the middle term. In this example, humans is the middle term.

Major premise: All mortals die.

Minor premise: All men are mortal.

Conclusion: All men die.

Here, the major term is die, the minor term is men, and the middle term is mortals.

Let us explore syllogism with some examples.

Two statements are given below followed by two conclusions numbered as I and II respectively. Consider the given statements as true even if they seem to be not. After reading all the conclusions, confirm which of the given conclusions logically follows, disregarding commonly known facts.

Q 1 - Statements:

I. Some goats are vegetarians.

II. All vegetarians are blessed.

Conclusions:

I. Some goats are blessed.

II. At least some blessed are vegetarians.

A - only conclusion I follows.

B - only conclusion II follows.

C - either conclusion I or II follows.

D - neither conclusion I nor II follows.

E - both conclusion I and II follow.

Answer : E

Explanation : It is given that some goats are vegetarians and all vegetarians are blessed. So we can say that some goats are blessed. Hence conclusion I follows. Also all vegetarians are blessed. Thus some blessed are vegetarians. Hence conclusion II follows.

Q 2 - Statements:

I. Some pictures are beds.

II. All beds are trees.

Conclusions:

I. Some pictures are trees.

II. At least some trees are beds.

A - only conclusion I follows.

B - only conclusion II follows.

C - either conclusion I or II follows.

D - neither conclusion I nor II follows.

E - both conclusion I and II follow.

Answer : E

Explanation : It is given that some pictures are beds and all beds are trees. Thus we can say that some pictures are trees. Hence conclusion I follows. Also all beds are trees. Thus some trees are beds. Hence conclusion II also follows.

Check your Progress-3

In each of the following questions two statements are given and these statements are followed by two conclusions numbered (1) and (2).

You have to take the given two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusions and then decide which of the given conclusions logically follows from the two given statements, disregarding commonly known facts.

Give answer:

- ❖ (A) If only conclusion (1) follows.
- ❖ (B) If only conclusion (2) follows.
- ❖ (C) If either (1) or (2) follows.
- ❖ (D) If neither (1) nor (2) follows.
- ❖ (E) If both (1) and (2) follow.

1. **Statements:** Some actors are singers. All the singers are dancers.

Conclusions:

1. Some actors are dancers.
2. No singer is actor.

2. **Statements:** All the harmoniums are instruments. All the instruments are flutes.

Conclusions:

1. All the flutes are instruments.
2. All the harmoniums are flutes.

3. **Statements:** Some mangoes are yellow. Some fruits are mangoes.

Conclusions:

1. Some mangoes are green.
2. Fruits are yellow.

4. **Statements:** Some ants are parrots. All the parrots are apples.

Conclusions:

1. All the apples are parrots.
2. Some ants are apples.

b) **Coding - Decoding**

Coding is a process used to encrypt a word, a number in a particular code or pattern based on some set of rules. Decoding is a process used to decrypt the pattern into its original form from the given codes. This process has tremendous application in data management, security, computing etc.

Coding-Decoding tests the student's ability to find out the rule under which a message has been coded and to decipher the message in the original form.

There are many types of challenges in coding and decoding. We will discuss few types here.

Type 1: letters

Example 1 : If DPT is EQU then what is TRY?

Explanation: If we observe carefully then we notice E is the letter next to D , Q next to P and U next to T, applying the same logic for TRY, we get the answer as USZ.

Ans1. USZ

Example 2: In a certain language, if BLOWN is coded as BLNOW then how will RIGHT be coded?

- a) HIRGT
- b) SJHIU
- c) GHIRT
- d) THIGR

Explanation: The letters of the word BLOWN are arranged in ascending order of English alphabets, similarly if we arrange the letters of the word RIGHT the arrangement will be GHIRT.

Ans2. c) GHIRT

Type 2: Numbers

Let us assign a number to each alphabet. Then each letter can be equated to a number as given below.

1	2	3	4	5	6	7	8	9	10	11	12	13
A	B	C	D	E	F	G	H	I	J	K	L	M
14	15	16	17	18	19	20	21	22	23	24	25	26
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Example 3: In a certain language, PAC is coded as 61, how will NEP be coded?

- a) 40
- b) 66
- c) 80
- d) 46

Explanation: If we arrange the English alphabets in reverse order then the positions of P, A and C are 11, 26 and 24 respectively. When we add these numbers we get 61. Similarly, when we add the reverse position numbers of the letters of the word PEN we get, $13+22+11$ i.e. 46.

Ans 3. d) 46

Type 3: Symbols

Example 4: If MADAM is coded as *?#?* and DOOM is coded as #%%* then LAD will be coded as

- a) ?#%
- b) &?#
- c) *#/
- d) ??#

Explanation: In this example we observe that letter M is represented by*, A by ?, D by # and O by % so LAD will be coded as &?#.

Ans 4. b) &?#

Example 5: If BRED is coded as #*?%, then BREEZE will be coded as

- a) #&%%?*
- b) ??%#*#
- c) %%%?^^
- d) ???%#@

Explanation: In this example we observe that the letters of the word BRED can be represented by any of the four symbols #,*,?, or %. When we observe the letters of the word BREEZE we find that E is repeating three times so the same symbol should appear thrice in the answer So we can zero down our answer to option c) or d) but no other letter is getting repeated in the word so all other symbols should be different. Hence our answer is ???%#@.

Ans 5. d) ???%#@

Check your Progress-4

1. In a certain language, if CAMEL is coded as DBNFM then how will ROOM be coded?

- a) NPPQ
- b) SPPN
- c) PPNON
- d) NQQP

2. If BLEPIN is coded as 987416, MATPIN is coded as 123416, then TABLE is coded as?

- a) 32987
- b) 32897
- c) 38987
- d) 21987

3. In a certain language,

'hi mi si' means 'Air is life'.

'si ni zi' means 'Balloon of Air'.

'ci zi mi' means 'Life of PI'.

Which of the following represents 'PI' in that language?

- a) hi
- b) mi
- c) ci
- d) si

4. In a certain language,

321 means 'Glass of Tea'.

426 means 'Tea is Brown'.

796 means 'Trunks are Brown'.

Which of the following represents 'is' in that language?

- a) 6
- b) 7
- c) 4
- d) 2

c) Blood Relation:

Finding out relationship between variables is very important in analyzing different situations. We can enhance this ability using the concept of Blood relations. Blood relation shows the different relations among the members of a family. Based on the information given, you are required to find the relation between particular members of the family.

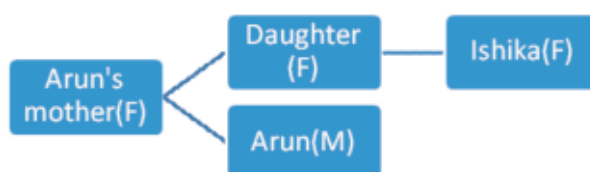
To solve the problems based on blood relation it is always better to draw the diagram or family tree on the basis of information given in the question. Each generation should be drawn at different levels of the diagram. These problems are based on blood relations strictly, so they do not include information about friends etc. Let's take a look at some important blood relations:

Father of father or mother	Grand father
Mother of father or mother	Grand mother
Brother of Father	Uncle
Sister of father	Aunt
Father of Wife/Husband	Father - in - law
Mother of Wife/Husband	Mother - in - law
Son / Daughter of Uncle/ Aunt	Cousin
Brother/Sister of Husband/Wife	Brother - in - law/Sister - in - law
Husband of Sister	Brother - in - law
Wife of Brother	Sister - in - law
Son / Daughter of Brother / Sister	Nephew/Niece
Brother/Sister of Mother	Maternal Uncle / Maternal Aunt

Please note that above table represents the relationship in English language. In other languages like Hindi there may be different words used for relationship. e.g in Hindi use of term uncle may not be similar to the usage in English.

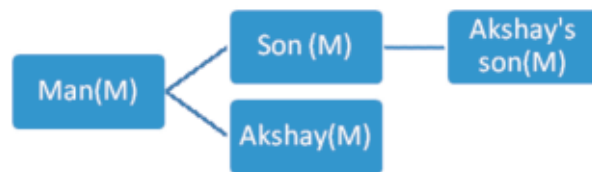
Example 1: Arun said, "Ishika is the daughter of my mother's daughter". How is Arun related to Ishika?

Explanation : Arun's mother's daughter is Arun's sister and her daughter Ishika is Arun's niece. Therefore, Arun is uncle of Ishika. If M stands for male and F stands for female, we can draw :-



Example 2: Pointing to a man, Akshay said, " His son is my son's uncle". How is the man related to Akshay?

Explanation: Akshay's son's uncle would be none other but Akshay's brother and his brother's father would be his father too. So, the man is Akshay's father.



Check your progress-5

1. Pointing to a picture the man said, "The lady in the picture is my nephew's maternal grandmother." How is the lady in the picture related to the man's sister who has no other sister?
 - a) Cousin
 - b) Sister-in-law
 - c) Mother
 - d) Mother-in-law
2. Meenakshi is Kirti's sister. Kaavya is Kirti's mother. Dipesh is Kaavya's father. Esha is Dipesh's mother. Then, how is Meenakshi related to Dipesh?
 - a) Grandfather
 - b) Grandmother
 - c) Daughter
 - d) Granddaughter

3. P+Q means P is the brother of Q, R-S means R is the father of S, S/T means S is the sister of T, T x U means T is the mother of U, which of the following means that O is the mother of N?

- a) L+M/N-O
- b) L-MxO/P
- c) N/MxL/O
- d) M+L/OxN

d) **Odd man out:**

For analysing a situation, one must be able to classify concepts or objects. In the classification one has to identify the common characteristics and distinguishing characteristics. Thus if we are able to enhance our skills in finding out the odd one out, we in turn will be able to enhance our analytical ability. In odd man out problems all the items given in the question except one follow a certain pattern or form a group. The item which is different and doesn't belong to that group will be the answer. Problems of this type are categorized under the head of 'classification'. That means out of all the given elements, one will not fall into the group due to some difference in the property. That is the odd element.

For Example -

Type 1 -

Which one of the words given below is different?

- a - Mango
- b - Apple
- c - Potato
- d - Cherry
- e - Blackberry

Explanation - Except potato, all the rest are the names of the fruits, while Potato is a vegetable. Hence, it is the odd man. So the answer will be (c)

Type 2 -

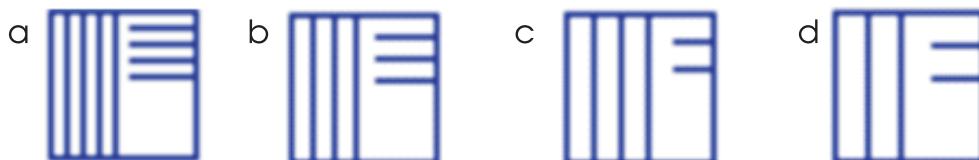
Find out the pair which is different.

- a - Cow and Buffalo
- b - Cock and Hen
- c - Horse and Mare
- d - Dog and Bitch
- e - Peacock and Peahen

From the above, the second one is the feminine of the first one except the first pair. So the answer will be (a).

Check your progress-6

1. Find the odd man out from the given alternatives.
 - a. Insurance
 - b. Provident Fund
 - c. Salary
 - d. Shares
2. Which number is odd man in the numbers 3, 5, 11, 14, 17, 21
 - a. 21
 - b. 17
 - c. 14
 - d. 3
3. Find out the odd man from the figures given below.



7.9 Practical work using spread sheet:

Practical-1: Demonstration of Not or negation

Use the NOT function, one of the logical functions, when you want to make sure one value is not equal to another.

Example

	A	B	C
1	Formula	Description	Result
2	=NOT(FALSE)	Reverses FALSE	TRUE
3	=NOT(TRUE)	Reverses TRUE	FALSE
4	=NOT(1+1=2)	Reverses TRUE	FALSE
5	=NOT(2+2=5)	Reverses FALSE	TRUE

Practical-2: Demonstration of 'and'

The AND Function returns TRUE if all conditions are true and returns FALSE if any of the conditions is false.

Example: Take a look at the AND function in cell D2 below.

D2		fx		=AND(B2>=60,C2>=90)					
	A	B	C	D	E	F	G	H	I
1	Name	Score 1	Score 2	Result					
2	Raghu	93	80	FALSE					
3	Jeet	60	91	TRUE					
4	James	58	75	FALSE					
5	Lina	79	94	TRUE					
6	Shalini	41	33	FALSE					
7									

Explanation: The AND function returns TRUE if the first score is greater than or equal to 60 and the second score is greater than or equal to 90, else it returns FALSE.

Practical-3: Demonstration of 'or'

The OR function returns TRUE if any of the conditions is TRUE and returns FALSE if all conditions are false.

Example : Take a look at the OR function in cell D2 below.

	A	B	C	D	E	F	G	H	I
1	Name	Score 1	Score 2	Result					
2	Raghu	93	80	TRUE					
3	Jeet	60	91	TRUE					
4	James	58	75	TRUE					
5	Lina	79	94	TRUE					
6	Shalini	41	33	FALSE					
7									

Explanation: The OR function returns TRUE if at least one score is greater than or equal to 60, else it returns FALSE.

Practical-4 : Demonstration of 'If'

The IF function checks whether a condition is met, and returns one value if true and another value if false.

Example: Take a look at the IF function in cell C2 below.

	A	B	C	D	E	F	G	H	I
1	Name	Score	Result						
2	Raghu	93	Pass						
3	Jeet	60	Pass						
4	James	58	Fail						
5	Lina	79	Pass						
6	Shalini	41	Fail						
7									

Explanation: If the score is greater than or equal to 60, then IF function returns Pass, else it returns Fail.

Summary

- A mathematically acceptable statement is a sentence which is either true or false.
- Negation of a statement p : If p denotes a statement, then the negation of p is denoted by $\sim p$.
- Compound statements and their related component statements:
A statement is a compound statement if it is made up of two or more smaller statements. The smaller statements are called component statements of the compound statement.
- A sentence with 'if p then q ' can be written in the following ways.
 - ❖ p implies q (denoted by $p \Rightarrow q$)
 - ❖ p is a sufficient condition for q
 - ❖ q is a necessary condition for p
 - ❖ p only if q
 - ❖ $\sim q$ implies $\sim p$
 - ❖ The contrapositive of a statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$.
- The converse of a statement $p \Rightarrow q$ is the statement $q \Rightarrow p$.
- $p \Rightarrow q$ together with its converse, gives p if and only if q .
- The following methods are used to check the validity of statements:
 - ❖ direct method
 - ❖ contra positive method
 - ❖ method of contradiction
 - ❖ using a counter example
- We can use logical reasoning in understanding statements, coding and decoding, in finding out relationship and classifying by finding odd man out.

Reference:

1. Mathematics textbook for class XI-NCERT

Solutions to Check your Progress

Check your Progress-1

Solutions: (Here statement means mathematically acceptable statement)

1. This sentence is false because 8 is greater than 6. Hence it is a statement.
2. This sentence is also false since there are sets which are not finite. Hence it is statement.
3. It is a scientifically established fact that sun is a star and, therefore, this sentence is always true. Hence it is a statement.
4. This sentence is subjective in the sense that for those who like mathematics, it may be fun but for others it may not be. This means that this sentence is not always true. Hence it is not a statement.
5. The sentence is a statement because it results in a true answer.
6. The sentence does not result in a true or a false answer so it is not a statement.

Check your Progress-2

Solutions:

7. Here "Or" is inclusive since school is closed on holiday as well as on Sunday.
8. Here "Or" is exclusive because it is not possible for two lines to intersect and be parallel together.
9. Here also "Or" is exclusive because a student cannot take both French and Sanskrit.

Check your progress-3

Solutions

1. A
2. B

3. D

4. B

Check your progress-4

Solutions

1. B

2. A

3. C

4. C

Check your progress-5

Solutions

1. C

2. D

3. D

Check your progress-6

Solutions

1. C

2. C

3. C